

Powell-Eyring Magneto-Nanofluid Flow over a Stretching Cylinder with n^{th} Order of Chemical Reaction

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Abstract

We considered MHD stagnation point powell-eyring magneto nanofluid over a stretching cylinder with double stratification and n^{th} order chemical reaction. A mathematical governing model has developed for the momentum, temperature and concentration boundary layer. Whereas this prominent transformation are used to transform the principal nonlinear boundary layer equations for momentum, thermal energy and concentration to a system of nonlinear ordinary coupled differential equations with fitting boundary conditions and are solved numerically by using finite difference method. The solution of Skin-Friction, heat transfer rate (Nusselt Number) and mass transfer rate (Sherwood number) are illustrated for the various important parameters entering into the problem separately are discussed with the help of graphs.

Keywords: *Nanofluid, Magneto hydrodynamics, Stagnation, chemical reaction.*

1. Introduction

Magneto hydrodynamics (MHD) concerns with the mathematical and physical scaffold that introduces magnetic-dynamic in electrically conducting fluids (eg. Plasams and liquid metals). Magneto hydrodynamics (MHD) incompressible viscous flow has many applications in science and engineering involving heat and mass transfer under the influence of chemical reaction and this frequently occurs in agriculture ,engineering, plasma studies and petroleum industries. The term nanofluid has been foremost introduced by Choi.

This nanofluid have been used potentially in numerous applications in heat and mass transfer, as well as microelectronics, fuel cells, pharmaceutical

sections and hybrid-powered engines, engine cooling/vehicle thermal management etc.

In outlook of applications, Abbasi et al. [1] investigated the doubly stratified mixed convection flow of Maxwell nanofluid with heat generation/absorption. Akbar et al. [2] magnetic field effects on Eyring-Powell fluid flow towards a stretching sheet of numerical analysis. Bilal et al. [3] studied the dissipative slip flow along heat and mass transfer over a vertically rotating cone by way of chemical reaction with Dufour and Soret effects.

Dogonchi and Ganji et al.[4] Investigation of MHD nanofluid flow and heat transfer in a stretching /shrinking convergent/divergent channel considering thermal radiation. Hayat et al. [5] explored Thermal Radiation Effect in MHD Flow of Powell-Eyring Nanofluid Induced by a Stretching Cylinder .Hayat et al. [6] studied the Influence of magnetic field in three-dimensional flow of couple stress nanofluid over a nonlinearly stretching surface with convective condition Hayat.et al.[7] experimentally the Effectiveness of magnetic nanoparticles in radiative flow of eyring powell field. Kaladhar et al.[8] has showed Double stratification effects on mixed convection flow of couple stress fluid in a non-Darcy porous medium with heat and mass fluxes. Khalil et al. [9] investigated the Numerical analysis for MHD thermal and solutal stratified stagnation point flow of Powell-Eyring fluid induced by cylindrical surface with dual convection and heat generation effects. Muhammad et al [10] analyzed MHD Boundary Layer Slip Flow and Heat Transfer of Ferrofluid along a Stretching Cylinder with

Prescribed Heat Flux. Ramzan et al. [11] studied the Radiative and Joule heating effects in the MHD flow of a micropolar fluid with partial slip and convective boundary condition. Ramzan [12] investigated the influence of Newtonian heating with viscous dissipation and joule heating on three dimensional MHD flow of couple stress nanofluid. Ramzan et al. [13] studied Radiative flow of powell-eyring magneto-nanofluid over a stretching cylinder with chemical reaction and double stratification near a stagnation point. Shehzad et al. [14] analyzed Boundary layer flow of third grade nanofluid with Newtonian heating and viscous dissipation.

In the present analysis, we have extended the work of Muhammad Ramzan et al. [15] for MHD stagnation point powell-eyring magneto nanofluid over a stretching cylinder with nth order chemical reaction. The governing equations are transformed into non linear ordinary differential equations and solved numerically using finite difference method.

2. Mathematical Formulation

Consider Eyring-powell nanofluid flow past a stretching cylinder with double stratification, thermal and magneto hydrodynamic. This study also considers nth order chemical reaction near a stagnation point. The cylindrical coordinates with z axis are along the stretching cylinder whereas r-axis upright to it.

We have

$$\tau_{ij} = \mu \frac{\partial w_i}{\partial z_j} + \frac{1}{\beta} \sinh^{-1} \left(\frac{1}{\lambda} \frac{\partial w_i}{\partial z_j} \right) \quad (1)$$

The governing equations are given by

$$\begin{aligned} \frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} &= 0 \quad (2) \\ u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} &= w_e \frac{dw_e}{dz} + v \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + \frac{1}{\rho \beta C_1} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \\ &- \frac{1}{6 \rho \beta C_1^3} \left(\frac{1}{r} \left(\frac{\partial w}{\partial r} \right)^3 + 3 \left(\frac{\partial w}{\partial r} \right)^2 \left(\frac{\partial^2 w}{\partial r^2} \right) \right) - \frac{\sigma B_0^2 (w - w_e)}{\rho} \quad (3) \end{aligned}$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{k}{\rho C_p} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{1}{\rho C_p r} \frac{\partial}{\partial r} (r q_r) r \left[D_B \frac{\partial T}{\partial r} \frac{\partial C}{\partial r} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial r} \right)^2 \right] + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad (4)$$

$$u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} = D_B \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) + \frac{D_T}{T_\infty} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - R_r (C - C_\infty)^n \quad (5)$$

The boundary conditions are

$$\begin{aligned} w &= \frac{U_0 z}{l}, u = 0, T = T_w = T_0 + a \left(\frac{z}{l} \right), C = C_w = C_0 + c \left(\frac{z}{l} \right) \text{ at } r = R \\ w \rightarrow w_e &= \frac{V_0 z}{l}, T \rightarrow T_\infty = T_0 + b \left(\frac{z}{l} \right), C \rightarrow C_\infty = C_0 + d \left(\frac{z}{l} \right) \text{ as } r \rightarrow \infty \quad (6) \end{aligned}$$

Where u and w are fluid velocity components along r and z directions respectively. The non-dimensional parameters β and c, b and d, $U_0, l, v, \rho, C_p, k, T, T_\infty, w_e, D_B, D_T$ are fluid parameters, dimensionless constants, reference velocity, characteristic length, kinematic viscosity, density, specific heat, thermal conductivity, fluid temperature, ambient temperature, stretching velocity, Brownian diffusion coefficient and thermophoretic diffusion coefficient respectively.

The Radiative heat flux term by using The Rosseland approximation is given by

$$q_r = - \frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial z} \quad (7)$$

Eq (4) becomes

$$\begin{aligned} u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} &= \frac{k}{\rho C_p} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{1}{\rho C_p r} \frac{4\sigma^*}{3k^*} \frac{\partial}{\partial r} \left(r \frac{\partial T^4}{\partial r} \right) \\ &\tau \left[D_B \frac{\partial T}{\partial r} \frac{\partial C}{\partial r} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial r} \right)^2 \right] + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad (8) \end{aligned}$$

We use the following similarity transformation,

$$\begin{aligned} \eta &= \sqrt{\frac{U_0}{vl}} \left(\frac{r^2 - R^2}{2R} \right), \psi = \sqrt{\frac{vU_0}{l}} R z f(\eta), \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \\ w &= \frac{U_0 z}{l} f'(\eta), u = -\sqrt{\frac{vU_0}{l}} \frac{R}{r} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (9) \end{aligned}$$

The equations (3),(5),(6),(8) becomes

$$(1+2\gamma\eta)(1+M)f'' + ff'' - (f')^2 + 2\gamma(1+M)f' - \frac{4}{3}(1+2\gamma\eta)M\gamma f'^3 - (1+2\gamma\eta)^2 \lambda M f'^2 f'' - Ha(f' - P_0) + P_0^2 = 0 \quad (10)$$

$$(1+2\gamma\eta)\left(1 + \frac{4}{3}Rd\right)\theta' + 2\gamma\left(1 + \frac{4}{3}Rd\right)\theta' + Pr(f\theta' - f'\theta - f'e) + Pr Nb(1+2\gamma\eta)\left(\theta'\phi' + \frac{Nt}{Nb}\theta'^2\right) + Q\theta = 0 \quad (11)$$

$$(1+2\gamma\eta)\left(\phi' + \frac{Nt}{Nb}\theta'\right) + 2\gamma\left(\phi' + \frac{Nt}{Nb}\theta'\right) + Pr Le(f\phi' - f'\phi - f'j) - Q_0\phi'' = 0 \quad (12)$$

$$f(0) = 0, f'(0) = 1, \theta(0) = 1 - e, \phi(0) = 1 - j, f'(\infty) \rightarrow P_0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0 \quad (13)$$

The non-dimensional parameters $\gamma, M, \lambda, Le, Pr, Ha, Q_0, Nb, P_0, Nt$ are curvature parameter, fluid parameters, Lewis number, Prandtl number, Hartmann number, chemical reaction parameter, Brownian motion, velocity ratio and thermophoresis parameter respectively.

$$\gamma = \left(\frac{\nu l}{U_0 R^2}\right)^{1/2}, Pr = \frac{\mu c_p}{k}, Le = \frac{\alpha}{D_B}, Ha = \frac{\sigma B_0^2 l}{\rho U_0}, M = \frac{1}{\mu \beta C_1}, \lambda = \frac{U_0^3 z^2}{2l^3 C_1^2 \nu}, P_0 = \frac{V_0}{U_0}, Rd = \frac{4\sigma^* T_\infty^3}{kk^*}, e = \frac{b}{a}, Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu}, Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}, j = \frac{d}{c}, Q_0 = \frac{R_r l}{U_0} \quad (14)$$

The skin friction, local Nusselt and Sherwood numbers are defined as follows:

$$C_f = \frac{\tau_{rz}}{\rho w_e^2}, Nu = \frac{z q_w}{k(T_w - T_\infty)}, Sh = \frac{z j_w}{k(C_w - C_\infty)}, \tau_w = \left[\left(\mu + \frac{1}{\beta c} \right) \left(\frac{\partial w}{\partial r} \right) - \frac{1}{6\beta C_1} \left(\frac{\partial w}{\partial r} \right)^3 \right]_{r=R}, q_w = -k \left(\frac{\partial T}{\partial r} \right)_{r=R}, j_w = - \left(\frac{\partial C}{\partial r} \right)_{r=R} \quad (15)$$

The non-dimensional forms are

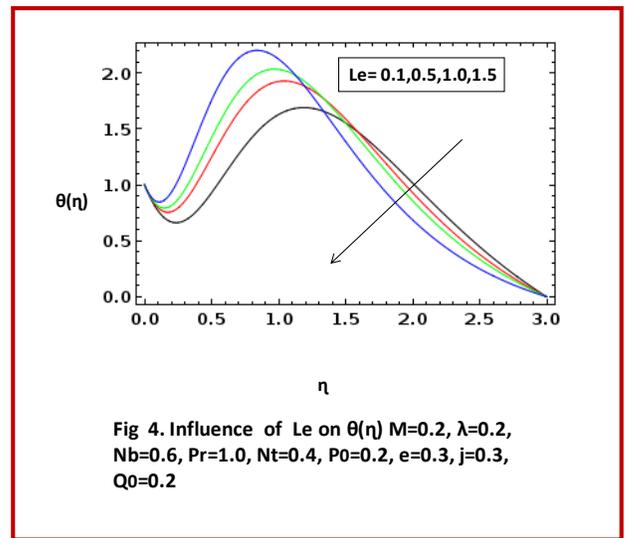
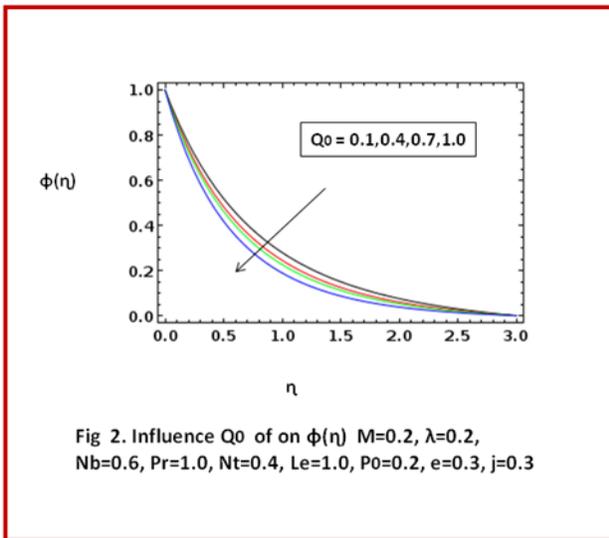
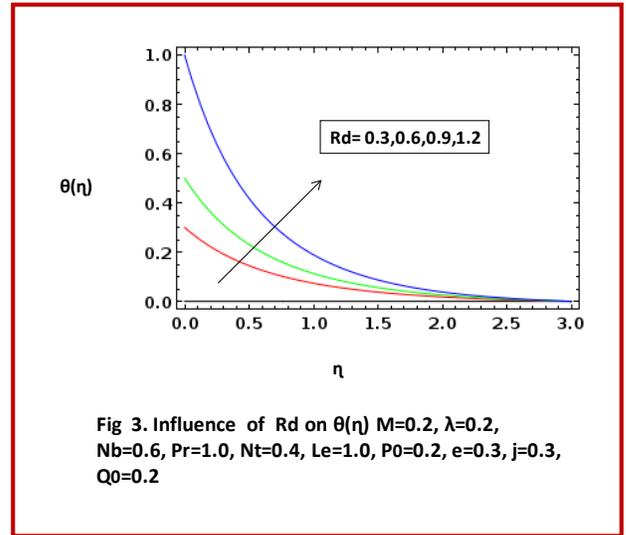
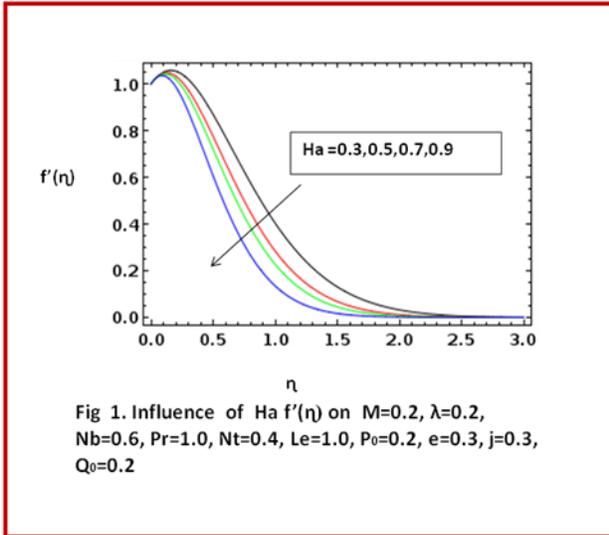
$$C_f Re_z^{1/2} = (1+M)f''(0) - \frac{\lambda}{3} M f'^3(0),$$

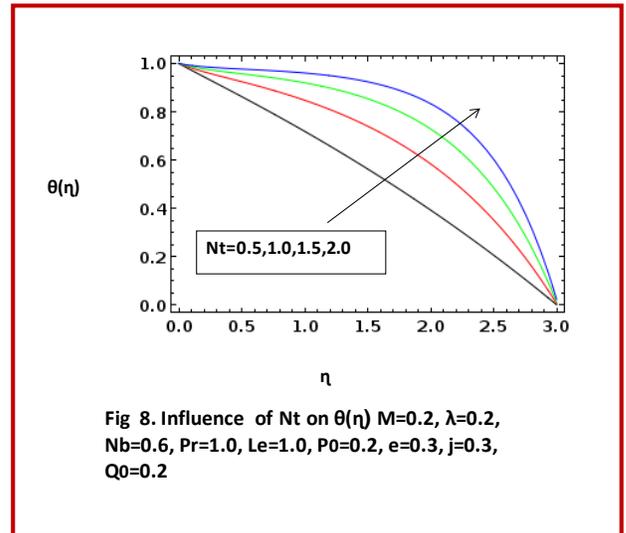
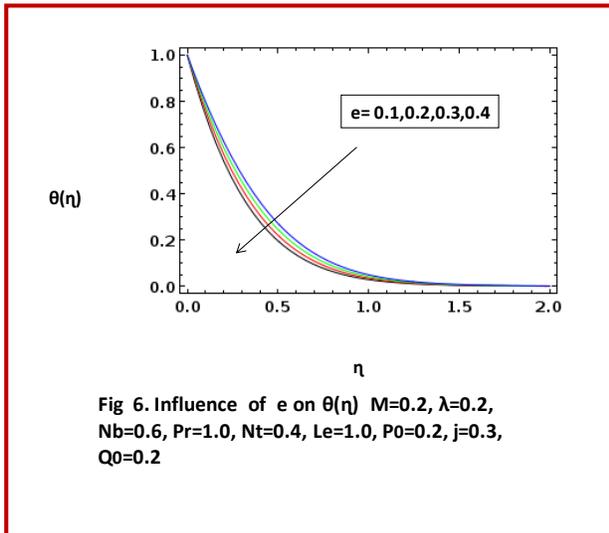
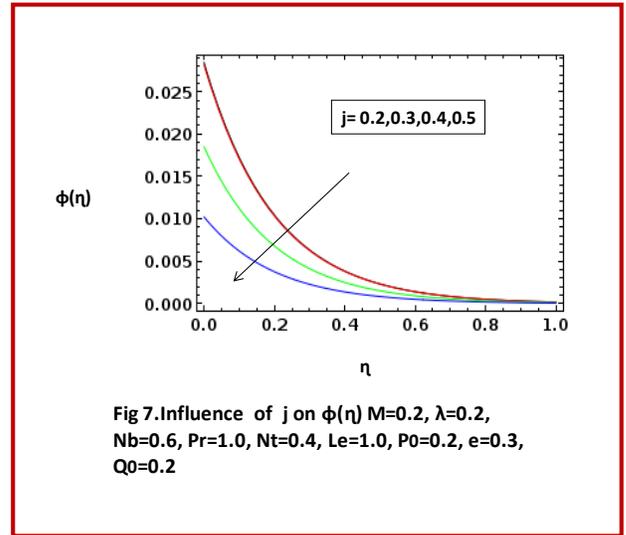
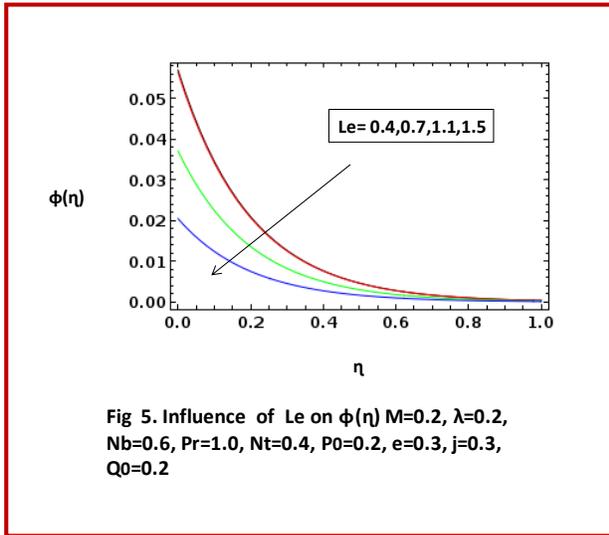
$$Nu Re^{-1/2} = - \left(1 + \frac{4}{3} Rd \right) \theta'(0),$$

$$Sh Re^{-1/2} = - \phi'(0) \quad (16)$$

3. Results and discussion

The results obtained for velocity, temperature and concentration distributions are presented graphically for using physical parameters. Moreover, graphical illustrations depicting impact of prominent parameters on skin friction, local Nusselt and Sherwood numbers are also added to the present exploration. From Fig 1, it is perceived that the velocity profile is decrease in diminishing function of Hartmann number Ha . Fig 2 illustrate that the concentration field for varied values of chemical reaction parameter Q_0 . Decrease in solute nanoparticle concentration. Fig 3 examined that the influence of radiation parameter Rd on temperature profile. Figs 4 and 5 depict the impact of Lewis number Le on temperature and nanoparticle concentration fields respectively and show that both profiles are decreasing functions of Le . Fig 6 demonstrates that thermal stratification e , temperature distribution also show a tendency to decline. The same fact holds in case of solutal stratification j and can be observed in Fig 7 where concentration profile is also the decreasing function of solutal stratification. in Figs 8 and 9 are illustrated the effects of thermophoresis parameter Nt on temperature and nanoparticle concentration respectively and the both distributions are mounting functions of Nt . Figs 10 and 11 shows that the increasing values of Pr results in reducing the temperature and concentration profiles. Figs 12 and 13 examined that larger values of Pr result because of smaller thermal diffusivity which ultimately lowers both temperature and concentration fields. Fig 14 and 15 represent the influence of Hartmann number Ha and fluid parameter M on skin friction coefficient. It is clear from the figure that skin friction coefficient is increasing function of both Ha and M . In Fig 16 and 17 are displayed the effects of thermophoresis Nt and Brownian motion Nb parameters on local Nusselt number. It is detected that increasing the values of Nt and Nb , results in lowering local Nusselt number.





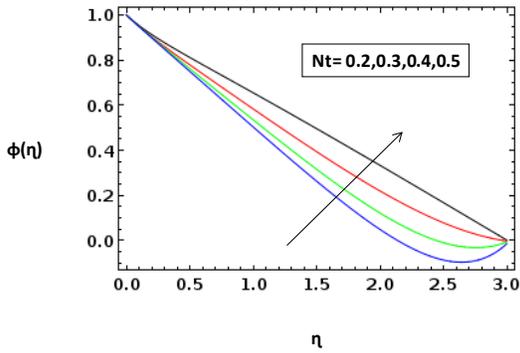


Fig 9. Influence of Nt on $\phi(\eta)$ $M=0.2, \lambda=0.2, Nb=0.6, Pr=1.0, Le=1.0, P_0=0.2, e=0.3, j=0.3, Q_0=0.2$

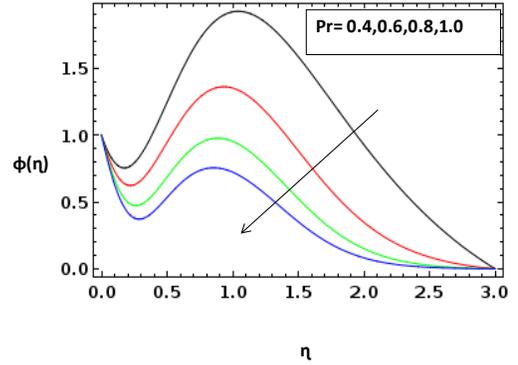


Fig 11. Influence of Pr on $\phi(\eta)$ $M=0.2, \lambda=0.2, Nb=0.6, Nt=0.4, Le=1.0, P_0=0.2, e=0.3, j=0.3, Q_0=0.2$

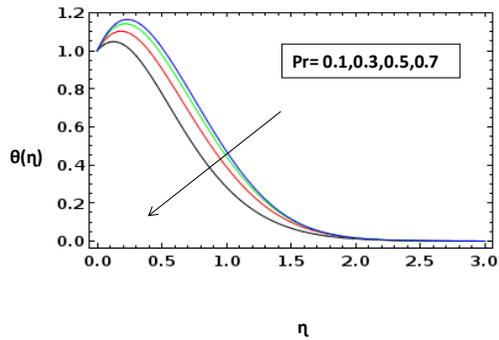


Fig 10. Influence of Pr on $\theta(\eta)$ $M=0.2, \lambda=0.2, Nb=0.6, Nt=0.4, Le=1.0, P_0=0.2, e=0.3, j=0.3, Q_0=0.2$

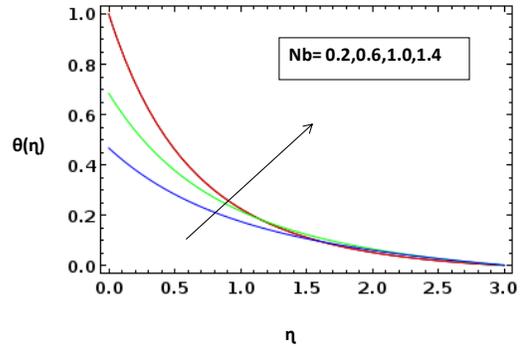


Fig 12. Influence of Nb on $\theta(\eta)$ $M=0.2, \lambda=0.2, Pr=1.0, Nt=0.4, Le=1.0, P_0=0.2, e=0.3, j=0.3, Q_0=0.2$

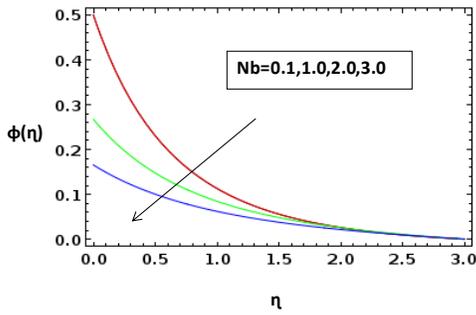


Fig 13. Influence of Nb on $\phi(\eta)$ $M=0.2, \lambda=0.2, Pr=1.0, Nt=0.4, Le=1.0, P0=0.2, e=0.3, j=0.3, Q0=0.2$

$$f''(0) - \frac{M\lambda}{3} f''^2(0)$$

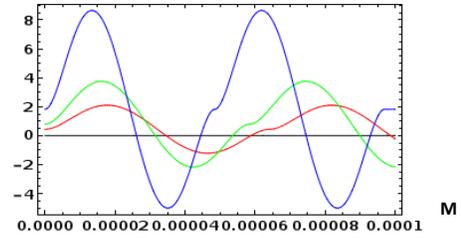


Fig 15. Influence of M on $C_f Re^{\frac{1}{2}}$
 $M=0.2, \lambda=0.2, Nb=0.6, Pr=1.0, Nt=0.4, Le=1.0, P0=0.2, e=0.3, j=0.3, Q0=0.2$

$$f''(0) - \frac{M\lambda}{3} f''^2(0)$$

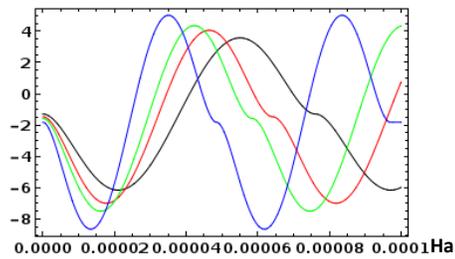


Fig 14. Influence of Ha on $C_f Re^{\frac{1}{2}}$
 $M=0.2, \lambda=0.2, Nb=0.6, Pr=1.0, Nt=0.4, Le=1.0, P0=0.2, e=0.3, j=0.3, Q0=0.2$

$$-\left(1 + \frac{4}{3} Rd\right) \theta'(0)$$

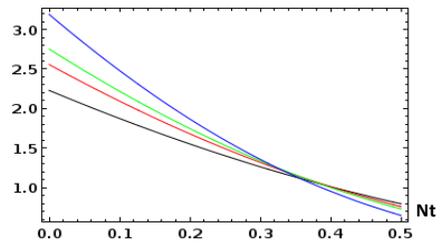
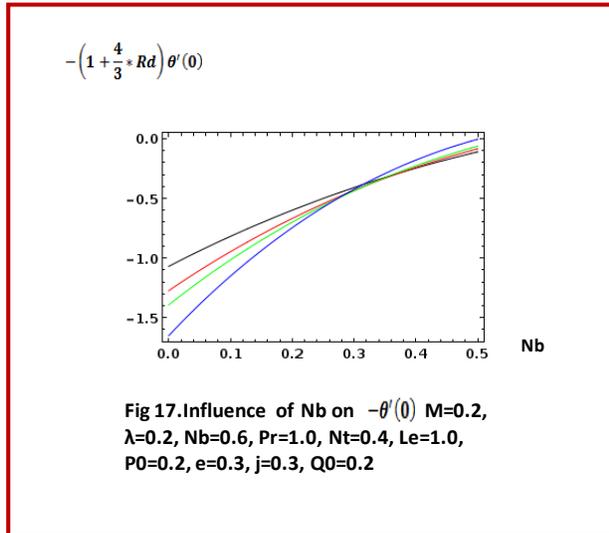


Fig 16. Influence of Nt on $-\theta'(0)$
 $M=0.2, \lambda=0.2, Nb=0.6, Pr=1.0, Nt=0.4, Le=1.0, P0=0.2, e=0.3, j=0.3, Q0=0.2$



4. Conclusion

In this study, a mathematical governing model has developed for the momentum, temperature and concentration boundary layer. Whereas this prominent transformations are used to transform the principal nonlinear boundary layer equations for momentum, thermal energy and concentration to a system of nonlinear ordinary coupled differential equations with fitting boundary conditions. The solution of Skin-Friction, heat transfer rate (Nusselt Number) and mass transfer rate (Sherwood number) are illustrated for the various important parameters entering into the problem separately are discussed with the help of graphs.

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